

# On The Pricing of Market-Makers Seats

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## Abstract

The purpose of our paper is to develop a contingent-claim analysis allowing for the valuation of organized exchange memberships or seats. Relying on the Mertonian methodology and in particular on Margrabe options, our model prices seats as two uncorrelated components: a basis value common to all seats and a personal value which takes into account the signalling mechanism arising on some peculiar exchanges.

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## Introduction

Organized exchange seats or memberships are valuable assets. They can be bought and sold, usually by means of an auction market, and partly represent a residual claim on the exchange itself. Like any assets, seats can be valued in different ways. In this paper, we concentrate on the Mertonian valuation of these property rights, and aim at giving a general methodology that can apply to the different types of auction markets currently existing.

Seats traded on the NYSE are by far the most famous ones. The literature on seats, shrunk to the extreme as can be perceived from the bibliography, essentially concentrates on the NYSE membership system. Nearly thirty years ago, the celebrated paper of Schwert [1977] paved the way for the analysis of seats value determinants and relationship to overall market conditions. In this article, Schwert elaborates on the statistical properties of NYSE seats prices, shows that they can be related to the total volume and market value of assets traded on the exchange, and claims that the multiplicative random walk (hence a lognormal process) is the good choice to depict seats price dynamics. NYSE seats trade on an anonymous auction market. They are limited in number since the creation of the exchange. In 1869, there were 1060 seats outstanding. Between that time and 1953, this number has only slightly increased. It has remained constant, at 1366, since then.

In the history of the NYSE seats market, a great period has been the economic expansion of 1925-1929. At that time, with the booming of the market, the high number of order executions to be fulfilled, and the need for more workforce, a great controversy emerged. The need was clear for an increase in the number of seats but many thought that an enlarged membership would mean a diluted value of their position. In 1925, members voted not to increase the number of seats. In 1928, President Simmons came back with a new proposal: the number of seats could be increased upon issuance of quarter dividend seats (each member would receive the quarter of a seat and could sell it freely provided it be in the next three years). The appropriateness of the suggestion, joined to the huge need for an increase in seats, led brokers to vote in the augmentation of the membership. A detailed account of this story can be found in Davis, Neal and Young [2003].

The acquisition of a seat offers diverse action possibilities. A classical distinction is made between specialists, commission brokers, floor or two-dollar brokers and floor traders. A specialist holds inventories of NYSE listed securities. A commission broker is employed by a brokerage house and

executes the orders of his firm's customers. A two-dollar broker executes orders for other exchange members. A floor trader executes trades for his own personal account. It has to be stressed that the buying of a seat leaves the buyer with the full choice to settle in any of these jobs.

A recent study extending Schwert's results can be found in Keim and Madhavan [2000]. These authors bring confirmation that seats prices can be regarded as a proxy for the NYSE market sentiment. In particular, the abnormal increase in seats value before 1929 and 1987 can be seen as an indicator of the bubble effects occurring during these periods. Keim and Madhavan also show that the seats market activity can be used to predict the excess returns of the *S&P* 500 index. Quoting their conclusion: 'seats prices do indeed contain important information about the beliefs of traders regarding future stock market activity, but this information is subtle and complex in nature'. Though memberships are similar to standard securities or assets, they are *not* exactly securities, as we shall detail in the body of this article.

Futures market seats represent a good instrument to improve the power and implement the influence of a member trading in a futures market, as explained by Paris [2000]. Bhasin and Brown [1994] point out the fact that in a futures market a seat can be regarded as a collateral inducing the member not to renege from trading. This is important in establishing the good reputation of the market member, otherwise referred to as 'local'. This is also the reason why the success of a seat's trade in a futures market is intimately related to the knowledge of the bidder identity in terms of quality and reputation. Each member interested in buying new seats is prompted to signal his own ability in trading by activating a mechanism allowing him to separate from traders whose gains are simply the consequence of superior information. Such considerations support our conjecture that the individual talent of locals is a crucial element in pricing seats in futures markets. Therefore, by assuming an auction mechanism which is not anonymous in assigning futures seats, we will try to define the seat's value in a way accounting for such an individual component.

This article will be devoted to the construction of a valuation model for seats prices. As a first step, we shall propose a general model that deals with the computation of the basis value-added inherent to all kinds of seats. This basis or *objective* value-added will be broken up into two parts: a part for the property right attached to the seat and a part for the trading gains that can be obtained from using the seat. This main component of

the seat's value will be priced as an annuity directly related to the trading gains, hence to the overall market volume and security prices. As a second step, in the next section, we will concentrate on the personal or *subjective* value that can be attached to seats in the particular case where the auction market is not anonymous. This personal value will be priced as a Margrabe option on the bidder and offerer brokers talents. The difference between the fair seat's value and its market price will be outlined. A final section will be dedicated to a numerical analysis of the financial implications stemming from our model.

## 1 Objective Value of a Seat

We open this section by giving the definitions and making the assumptions underlying the whole body of our paper; then, we move on to the computation of the objective value-added of the seat. Finally we will conclude the section by doing a general sensitivity analysis of the above-mentioned value-added.

### 1.1 Assumptions and definitions

Let's start by assuming that the objective value of a seat is the sum of two components: a property right, dependent on the kind of market where the seat is traded on, and a second component reflecting the claim of the seat-holder on expected future cash flows produced by trading activity run on the seat. We shall mainly concentrate in this section on the second component because it reflects the biggest part of the value embedded in a seat.

Indeed, when a market-maker buys a seat, he pays a certain amount that allows him to receive cash-flows for as long as he trades; then, he can sell the seat and receive an additional cash-flow reflecting the value of the sale. By *objective value-added* (simplified in *objective value* very often in the text of this article), it is clearly meant the discounted value of the future cash-flows arising from trading through the seat, explicitly excluding the final positive cash-flow corresponding to the sale of the seat. As these cash-flows are obtained by taking positions on market instruments, we assume the existence of possible *ad hoc* hedging strategies and therefore price this contribution to the price of a seat under the risk-neutral world, noticeably discounting with the risk-free rate in such a universe.

As to the first component, it certainly depends on variables measuring how important the market, where the seat-holder is trading, is. Think for instance of the number  $N_t$  of securities exchanged in the market (trading volume) or of the average price  $P_t$  of traded securities on the Exchange. Therefore, we indicate in general terms:

$$I_t = I(N_t, P_t)$$

as the indicator of the seat's residual value. Dealing with the shape of such a function is a hard matter. Moreover, we would have to consider that it could be also interrelated, to some extent, with the expected cash flow component. For the sake of simplicity, in a quite realistic way, we will consider such a quantity as an exogenously given amount whose value depends on time:

$$I_m(t) = \bar{I}(t)$$

that is attached to the seat's global value, depending on the reputation of the Exchange where the membership is exercised.

Let us now move on to the modeling of the main component of the seat's value, which is the one related to the expected cash flow perceived by the seat-holder as a consequence of his/her trading activity. To this respect we must recognize that market makers on a given Exchange have individual features making them different one another. Therefore, in order to compute an objective value, we must borrow from the utility theory the concept of "representative agent", which is the representative market maker, in this case. He/she has to be interpreted as the one adequately representing the common features of all the market-makers acting on the Exchange. All the quantities defined in the sequel of this subsection must be referred to the representative market maker.

Let's start by modeling the dynamics of the gross profit rate per contract traded by the representative seat-holder, or  $p$ , as a lognormal motion like:

$$dp_t = \mu_t^p dt + \sigma_t^p dz_t^p \tag{1}$$

where  $\mu_t^p = \mu^p p_t$  and  $\sigma_t^p = \sigma^p p_t$ .

Let also the number of contracts traded by our representative local be defined by  $n$ , described here again by a lognormal process:

$$dn_t = \mu_t^n dt + \sigma_t^n dz_t^n \tag{2}$$

where  $\mu_t^n = \mu^n n_t$  and  $\sigma_t^n = \sigma^n n_t$ .

The gross profits earned by the representative local, defined by  $\pi = p n$ , follows according to Itô's lemma:

$$d\pi_t = \mu_t^\pi dt + \sigma_t^{\pi,1} dz_t^n + \sigma_t^{\pi,2} dz_t^p$$

where

$$\mu_t^\pi = \mu_t^n p_t + \mu_t^p n_t + \rho_{n,p} \sigma_t^n \sigma_t^p$$

and

$$\sigma_t^{\pi,1} = p_t \sigma_t^n = p_t n_t \sigma^n$$

and

$$\sigma_t^{\pi,2} = n_t \sigma_t^p = p_t n_t \sigma^p$$

and  $\rho_{n,p}$  is the correlation between the two driving Brownian motions defined upwards.

Now it is important to consider that operative costs intervene and diminish the flow of benefits entitled to the local. Similarly as in any company, there are fixed and variable costs affecting the business. We assume that the variable costs are proportional to the number of contracts traded and model the operative costs  $c$  by the following sum of two contributions:

$$c_t = a + b n_t$$

Therefore:

$$dc_t = \mu_t^c dt + \sigma_t^c dz_t^n$$

where  $\mu_t^c = a + b \mu_t^n$  and  $\sigma_t^c = b \sigma_t^n$ .

## 1.2 Towards a seat's Objective Value

A naive market value of the seat's added value would be given by taking the risk-neutral expectation of the properly discounted trading gains (profits minus costs), yielding the perpetuity:

$$S_t = E_Q \left( \int_t^{+\infty} (\pi_s - c_s) e^{-r(s-t)} ds \right)$$

where by  $S$  is meant the seat's value.

However, this is a bit simplistic and two supplementary elements should be taken into account in order to get the correct formula. First of all, any trader has a limited lifetime and he is only interested in the expected value of what he can definitely get in the future. Though cashflows will continue to accrue after the death of the local (and assuming he keeps his seat until death), this is of no interest to him, but rather to the next local who will take the continuation of the seat. Then, why should the value of the seat include the future cashflows accruing in fifty or seventy years? Market fair valuation would classically include and discount all the future cashflows, but here, since we are talking of a position and *not* a security, we argue that the fair added-value should include all the future cash flows only up to the death of the local or the sale of the seat<sup>1</sup>. Remember here that we are dealing with an added-value; therefore the proceeds of the final sale will not be included during our discounting process.

Secondly, it should not be forgotten that a local may default during the exercise of his/her work. In particular, if the gross profits are not enough to compensate for the operative costs he incurs, he has to give up his seat. We assume that a collateral has been posted by the local in order to cover such losses but that it may not be sufficient to always meet the trader's obligations. We denote by  $\omega$  this collateral. Suppose then that at time  $t$  one has:

$$\pi_t + \omega < c_t$$

this would be a situation where default would occur and cashflows would stop accruing to the local. But of course, this is not the most general way default could happen. In particular, the collateral could be exhausted in a few times by covering successive minor losses. The general default time therefore becomes:

$$\inf \left\{ t \mid \int_0^t (c_s - \pi_s) ds > \omega \right\}$$

To conclude, we have to take into consideration two stochastic times ending the arrival of cashflows. We denote by  $\tau_1$  the first of these ; it is the time where natural handing back of the seat occurs, either by death or by free sale. Let then  $\tau_2$  denote the date of default. Finally,  $\tau = \min(\tau_1, \tau_2)$  is the time when net cashflows will stop accruing to the local.

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<sup>1</sup>Another good reason for handing back the exchange membership could be the seat-holder's retirement. For sake of tractability, however, we'll concentrate on death and free sale as possible reasons of a seat's abandonment

The terminal objective added-value from possessing the seat (neglecting the residual claim  $I_m = \bar{I}$ ) should therefore be at any given time  $t$ :

$$S_t = E_Q \left( \int_t^\tau (\pi_s - c_s) e^{-r(s-t)} ds \right) \quad (3)$$

where  $r$  is the risk-free interest rate. We now come to the practical implementation of this formula.

### 1.3 Implementing the model

Let us first develop formula (3). We get:

$$S_t = E_Q \left( \int_t^\tau (p_s n_s - (a s + b n_s)) e^{-r(s-t)} ds \right) \quad (4)$$

Now, we consider the simplified situation where profits are received each year and operative costs paid with the same frequency. This allows us to discretize the preceding equation. Of course, a finer description would also be possible by considering profits paid with a higher frequency, but this would be a straightforward generalization that would not change the sensitivities computed hereafter. The discretized version of equation (4) can therefore write as:

$$S_t = E_Q \left( \sum_{t_i=t}^{\tau} (p_{t_i} n_{t_i} - (a t_i + b n_{t_i})) e^{-r(t_i-t)} \right) \quad (5)$$

where  $t_i$  indicates a quantity referring to the year beginning at  $t_i - 1$  and ending at  $t_i$ .

For the sake of simplification, we assume that  $\tau_1$  is a constant and equal to the average time spent by a member on his/her seat - excluding in the historical estimation the few members who defaulted. To express that it is a constant, we write  $\tau_1 = T_1$ . Coming to  $\tau_2$ , it is the first time when the cumulated gross profits added to the collateral cannot cover up the cumulated operative costs. Thus, we have:

$$\tau_2 = \min \left\{ t_i \mid \sum_{j=0}^{t_i} (c_j - \pi_j) > \omega \right\}$$



Finally  $\tau$  can be developed as:

$$\tau = \min \left\{ T_1, \min \left\{ t_i \mid \sum_{j=0}^{t_i} (c_j - \pi_j) > \omega \right\} \right\} \quad (6)$$

Let us now come to the effective valuation of formulae (4) and (5). Formula (4) strongly resembles the one of an Asian option. Yet, this is a bit more complicated. First of all, the product  $p n$  appears: two correlated driving Brownian motions are part of the game. Secondly, time  $\tau$  does not translate in a simple condition that would simply be taken into account by writing  $(\pi_t - c_t)^+$  or by introducing a manageable indicator function. Also, one should not forget that all accrued benefits from the past are kept by the local in case of default.

The conclusion of the above discussion is that formula (5) should be computed by means of simulations. To ease this process, we decorrelate the two driving Brownian motions (note that this is not compulsory). Define therefore:

$$\begin{cases} d\hat{z}_t^p &= \frac{1}{\sqrt{1-\rho_{n,p}^2}} dz_t^p - \frac{\rho_{n,p}}{\sqrt{1-\rho_{n,p}^2}} dz_t^n \\ d\hat{z}_t^n &= dz_t^n \end{cases}$$

It can be easily checked that  $E_Q(d\hat{z}_t^p \cdot d\hat{z}_t^n) = 0$ . Now the two original Brownian motions express in terms of the ones just introduced as:

$$\begin{cases} dz_t^p &= \sqrt{1-\rho_{n,p}^2} d\hat{z}_t^p + \rho_{n,p} d\hat{z}_t^n \\ dz_t^n &= d\hat{z}_t^n \end{cases}$$

This allows us to rewrite our main dynamics (1) and (2), under the historical measure, according as:

$$\begin{cases} dp_t &= \mu_t^p dt + \sigma_t^p (\sqrt{1-\rho_{n,p}^2} d\hat{z}_t^p + \rho_{n,p} d\hat{z}_t^n) \\ dn_t &= \mu_t^n dt + \sigma_t^n d\hat{z}_t^n \end{cases}$$

By assuming the profit per contract to be hedgeable and similar to a security, we can change the probability writing under the risk-neutral measure  $Q$ :

$$\begin{cases} dp_t &= r dt + \sigma_t^p (\sqrt{1-\rho_{n,p}^2} d\hat{z}_t^p + \rho_{n,p} d\hat{z}_t^n) \\ dn_t &= \mu_t^n dt + \sigma_t^n d\hat{z}_t^n \end{cases} \quad (7)$$

To conclude, our program is to solve equation (5) where the annuity ends at the time given by (6) and assuming the dynamics above in (7).

## 1.4 Sensitivity Analysis

We are now in a position where we can lead a sensitivity analysis of the objective value of a seat with respect to the main determinants at stake. Unless specifically stated, we take the following range of parameters in this subsection:

$p_0$	$\sigma^p$	$n_0$	$\sigma^n$	$\mu^n$	$\rho_{n,p}$	$\omega$	$T_1$	$a$	$b$	$r$
1000	0.1	2520	0.05	0.1	0.5	1000000	15 years	1000000	600	0.03

Table 1: Model Parameters

The initial profitability per contract is 1 000€; the profitability per contract's volatility is assumed to be reasonably high and comparable to the one of a stock, we set it to 10%. The initial average number of contracts traded per year is taken equal to 2520, hence to about ten contracts per working day. We assume that its drift is of ten percent, meaning that the market-makers ability to trade profitable contracts increases progressively with time. The corresponding volatility is set to 5%, less than the one of the profitability per contract. We assume that this dynamics, which is not the one of a security or a profit mimicking a security, is quite highly correlated to the profit per contract dynamics, with a coefficient of 0.5.

A collateral of 1 000 000€ has been posted at inception of the seat. The average free seating period is set equal to fifteen years. The fixed costs incurred each year are 1 000 000€ whilst the proportionality coefficient between the yearly variable costs and the number of contract traded each year is equal to six hundred. Finally, the risk free interest rate is set to 3%.

The effect of increasing the collateral value at inception of the seat on the total value of the seat is plotted in figure 1. It appears clearly that increasing the collateral has first a beneficial effect but that beyond a numéraire of 3 000 000 €, the value of the seat increases slightly, and only because of the added value of the collateral. Note that we could have graphed the value of the seat without taking into account the collateral value and that it would have led to an optimum around the same value of 3 000 000 €. The

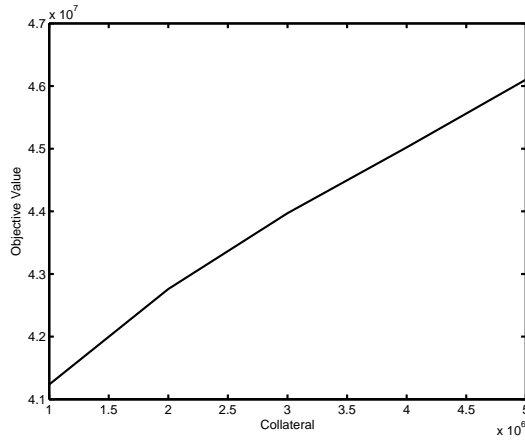


Figure 1: Objective Value w.r.t. Collateral

conclusion is clear: there is certain point beyond which the market-maker is enough protected and where there is no need for increasing the collateral posted at inception.

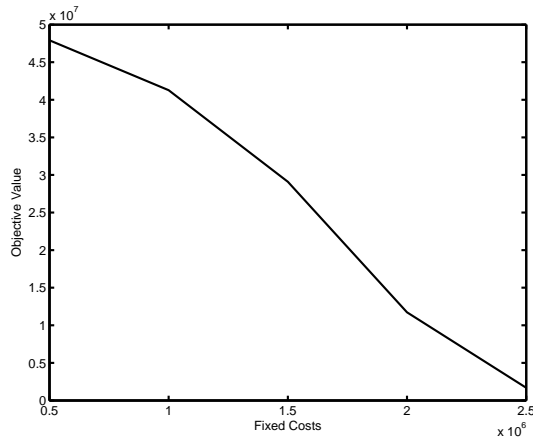


Figure 2: Objective Value w.r.t. Fixed Costs

In figure 2, we plot the seat value for fixed costs ranging from 500 000 to 2 500 000 € per year. Of course the higher the fixed operative costs the lower the value of the seat, due to two reasons: first, expected cash-flows decrease according to the costs, second, the probability of default also increases, and hence in some occasions the possibility to continue to benefit

from positive cashflows disappears. It appears clearly that beyond 1 000 000 € per year, the seat's value decreases sharply and the business is in danger.

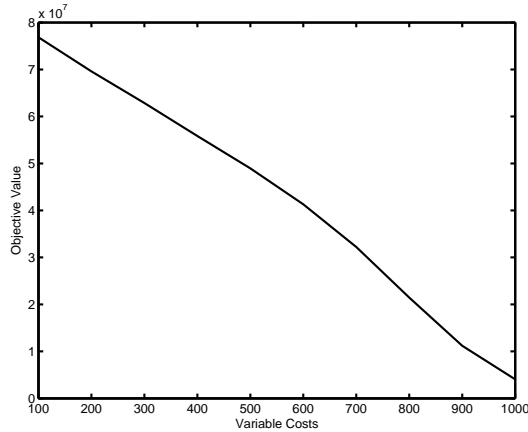


Figure 3: Objective Value w.r.t. Variable Costs

Then, in figure 3, we plot the seat value for variable costs ranging from 100 to 1 000 (this is indeed the proportionality coefficient multiplying the number of contracts to yield the variable costs). It can be observed that of course, the higher this coefficient, the lower the value of the seat ; yet, the effect of variable costs is less harsh than the one of the fixed costs.

## 2 Subjective Value of a Seat

The knowledge of the value of the net trading profits per broker entitles us to compute the subjective value attached to the seat in the case when the market is not anymore anonymous. This subjective value is simply a Margrabe option on the net expected trading profits  $\Psi^s$  and  $\Psi^b$ , or talents, of the seller ( $s$ ) and the bidder ( $b$ ).

### 2.1 Outline of the Problem

The most innovative part of our model consists of recognizing that the seat's value can incorporate a component depending on the specific talent of the persons exchanging the seat. Seats are usually traded by implementing an auction process. In those cases where the auction is not anonymous, the subjective part of the seat's value can be represented by the current value of a

Margrabe American style option written on the talents of the two exchanging locals. As to the maturity of such an option it seems to be correct defining it as in (6), having in mind that in this case the expression for  $\tau$  is referred to the seat potential seller instead of the representative local. This is the reason why we convene of indicating the option's maturity as  $\tau^s$ . In this way we capture also the effect of an expected seat's holding time which is different from the one characterizing the representative local.

By talents, we mean the net profits expected by each of the two traders at the time of the auction, as will become clear in the following. The quality indicator used can either be the average gross profit rate per contract or the number of contracts traded - or both. The average gross profit rate in turn depends on the securities' price, the bid-ask spread, and so on.

In mathematical terms, the seat's owner is long an American call expiring at  $\tau^s$  whose payoff at exercise is:

$$C_{t_e} = \max(\Psi_{t_e}^s - \Psi_{t_e}^b, 0) \quad (8)$$

where  $t_e$  is the exercise time whilst  $\Psi_{t_e}^s$  and  $\Psi_{t_e}^b$  are the net expected trading gains for the seller and *winning* bidder respectively at the time of the option exercise. The first term is the seller's underlying and the second one the call option's strike.

The call option defined by (8) can also be interpreted as a put option on the talent of the seat's bidder. In case the bidder is the one winning the auction process, the current value of (8), intended as a put, is strictly related to the seat's assignment price.

The main problem in valuing such a kind of option is related to the applicability of the Black and Scholes arbitrage argument to an underlying which is certainly not tradable. If arbitrage theory does not work in this case, a closed form solution to the valuation problem cannot be assigned and numerical valuation methods could be the only promising way of approaching the pricing of our 'embedded' option.

In figure 4, we display the plot of a typical Margrabe option done with Matlab - where both underlying assets are lognormal, and  $\rho$  is the correlation.

Assume  $U$  and  $V$  are the net expected trading gains of the seller and the bidder. Figure 4 can easily be construed in terms of membership

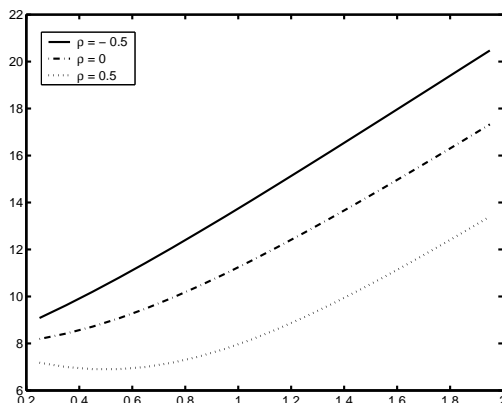


Figure 4: Option to buy  $V$  in exchange for  $U$  w.r.t.  $\sigma_V/\sigma_U$

implications. First of all, picking a single curve, it appears that more volatility means more value: one is interested in exchanging talents because the exchange of positions will mean a gain of profit volatility and therefore a gain in absolute terms (the higher  $\sigma_V$  with respect to  $\sigma_U$ , the higher the exchange option's value). Then, comparing curves, it appears that a negative correlation, close to  $-1$ , of the underlying talents, is more valuable to the bidder: for instance the curve with  $\rho = -0.5$  is located much above the one with  $\rho = 0.5$ . Let us now come to a more general analysis.

## 2.2 From the seat's value to the seat's price

Summarizing what has been done up to now, the most complete expression for the value of an Exchange membership is the following one:

$$V_t = \bar{I}(t) + S_t + C_t \quad (9)$$

The quantity in (9) expresses the fair value of the seat for the specific local according to his/her own features. Such a quantity, however, is not the one that the seat-buyer is available to pay. In fact, a seat's price equal to  $V_t$  would imply the Exchange membership being a zero-NPV investment, which is not the case for a local willing to make profits with his/her asset.

We could simply assume the local planning a constant period by period inflow from the seat's related operations given by  $W_{t_i} = W$ . In principle the expected present value of such an annuity would reflect the specific qualities of the market maker and the time spent on the seat as well.

To make our analysis as simple and general as possible consider the expected present value of the annuity by the representative local's point of view defined as follows:

$$\bar{W}_t = W \sum_{s=1}^{\tau-1-t} {}_sE_t \quad (10)$$

where  ${}_sE_t = 1/(1+i)^s {}_sq_t$ ,  $i$  is the one-period discounting rate and  ${}_sq_t$  is the probability that the local will still be on the seat  $s$  periods after the evaluation time  $t$ . Based on (10), the price paid by the local to buy the Exchange membership is:

$$P_t = \bar{I}(t) + S_t - \bar{W}_t \quad (11)$$

which is, obviously, a price accounting just for quantities independent of the locals' specific qualities.

Including subjective components into the quoted price requires, firstly, the computation of the seat's value by means of (9). Also, the annuity's value has to be adjusted according to the expected seat's holding time of the trader and his/her own specific talent, as well. Therefore, the quantity  $\bar{W}_t$  is replaced by  $\hat{W}_t$  which is a subjective one. It is computed by adjusting the probability  ${}_sq_t$  according to the local's personal expected seat's holding period and the discounting rate  $i$  according to his/her expected profit rate per traded contract<sup>2</sup>. Conclusively, the seat's price reflecting the local's specific qualities can be written as follows:

$$\hat{P} = \bar{I}(t) + S_t + C_t - \hat{W}_t \quad (12)$$

where all the quantities involved have already been previously defined.

### 2.3 The seller and bidder's viewpoints

Obviously, because of the subjective component, either the seat's value or the seat's price from the seller point of view are different from the ones of the bidder - unless the two have the same quality. This fact leads to three different problems:

1. Define the condition pushing the seat's owner to sell the seat

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<sup>2</sup>In particular, a lower discounting rate will correspond to a higher expected profit rate per contract

2. Define a mechanism allowing the bidder characterized by a high personal talent to separate from the one endowed with superior information
3. Define the conditions - to be verified at the end of the auction process - allowing the seat's trade to be executed

All the aforementioned problems can be indifferently solved in terms of the seat's value or the seat's price. Accordingly with the analysis developed in Paris (2000), we'll briefly deal with them referring to the seat's value.

As to point 1., Paris (2000) shows that the required condition is:

$$V_t^s > E_t(V_{t+\Delta t}^s) \quad (13)$$

implying that the seat's owner is available to sell his/her membership if he/she is expecting a reduction of the seat's value in the near future.

Item 2. involves a signalling problem arising because of the fact that locals with low skills but endowed with superior information could behave like high quality locals by bidding comparable prices. Therefore, the quality bidder has an incentive to separate from the informed one, in order to be sure of getting the seat. Such a problem is solved in Paris (2000) by the quality bidder posting a bid given by:

$$V_t^{qb-p} = V_t^{qb-f} - c_t^q \quad (14)$$

where it is meant that the posted bid must be equal to the difference between the quality bidder's fair seat's value and the present value of the signal's expected relative cost<sup>3</sup>. Paris (2000) shows that, under plausible conditions, such a bid allows the quality buyer to incur an effective cost which is equal to the fair seat's value plus the cost of signalling, while the informed buyer trying to do the same thing will incur a cost bigger than the seat's fair value. The costly signal consists of a reduction of the bid-ask spread quoted by the quality bidder with respect to trades performed during the auction time.

We give now the two viability conditions solving 3. and allowing for the execution of the trade at the end of the auction. Paris (2000) shows that such conditions must be verified because of the time elapsing between the bids' posting and the seat's assignment. This time interval could change

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<sup>3</sup>Obviously, in this case we are not considering the annuity to be subtracted to get the seat's price



the relative positions of the seat's trade counterparties. From the seller's point of view the condition is:

$$c_t^a > S_t^b - S_{t_1}^s \quad (15)$$

while from the bidder's point of view it is:

$$E_t(\tilde{c}_{t_1}^b) > c_{t_1}^b \quad (16)$$

Equation (15) means that with risk averse traders, the seat's trade is executed if and only if the buyer's seat reservation price at the time of the bid submission is higher than the seller's reservation price at the time of the seat's assignment, while the buyer's present value of the expected relative cost of signalling,  $c_t^a$ , must be bigger than such a difference. Then, equation (16) implies that the realized buyer's signalling relative cost has to be lower than its current expectation.

## Conclusion

We have addressed in this article some of the main problems arising when one is willing to give a value to a particular seat on a particular market. It has appeared that there is a multiplicity of possible treatments of such a protean and difficult problem. Indeed a market maker's seat is not exactly a tradable asset, but its value, direct consequence of a series of market inflows and outflows, is directly linked to the evolution of the market (as shown by the preceding authors) and therefore of its related securities.

In a first step, we have concentrated on the computation of the added-value due to trading through the seat. Indeed, a seat is a lifelong investment to most market actors. Investing in a seat is crucial because you also invest your time and occupations in it. Choosing to buy, or not, a seat, is choosing to an everyday activity and the access to a series of cash-flows. Therefore, due to this very important nature of seats, one would tend to be extremely cautious in the choice of such or such seat.

We have elaborated a simple model to price the series of cash-flows that can be earned through the possession of a seat. Our model postulates that the considered earnings, stemming from the trading of futures, bonds, or such instruments, are liable to hedging strategies, and that therefore they can be priced in market-value as a sum of payments discounted at the risk-free rate in the risk-neutral world. In particular, we end up with formulae where

the discounting process stops when trading is halted, due to retirement or default of the owning market-maker. Of course, this is an approach amongst possible other ones - our aim being here to furnish a method simple to implement, easy to construe, and coherent with the seats related contingent claims valuation procedures.

In a second and important step of our paper, we turn to the treatment of an important question: the treatment of the signaling mechanism that arises in some markets between seat sellers and bidders. Indeed, in such an archetypal game, what are the conditions that push sellers to indeed sell their seat, and what are underlying forces that drive bidders to distinguish themselves in terms of trading talent when some information asymmetries favor some market-makers with respect to others? Our treatment, based on Paris (2000), gives some hints, both qualitative and quantitative, that help understand and solve this problem.

As mentioned above, the task of modeling a seat's value is a difficult one. We have explored a few paths leading to a multifaceted understanding and solving of this problem. Of course, there is some room left for future research and explorations. In particular, an interesting challenge for a future article would be to concentrate on a particular market (for instance the futures one) and check the numerical adequacy of the approach suggested in the current article.

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